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Roll No ....

BE-3001(ME)-CBGS

B.E. IV Semester

Examination, December 2020

Choice Based Grading System (CBGS)

Mathematics - III

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Express $f(x) = \pi x - x^2$ as a half range sine series in $0 < x < \pi$.

b) Obtain Fourier series of the function:

$$f(x) = x\sin(x) \quad \text{when } -\pi < x < \pi$$

2. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$.

b) Find the inverse Laplace transform of function.

$$\frac{s}{\left(s^2 + a^2\right)^2}$$

3. a) Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function of this as the real part.

   b) State and prove Cauchy's Theorem.

4. a) Determine the poles and residue at each pole of the function

$$f(z) = \frac{z^2}{(z+2)(z-1)^2}$$

   b) Find the kind of singularities of the function

$$\frac{\cot \pi z}{(z-a)^2} \text{ at } z = a \text{ and } z = \infty$$

5. a) Use Euler's modified method to compute y for $x = 0.2$, given that $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0, y_0 = 1$ result correct upto three decimal places.

   b) Using Picard's process of successive approximations obtain a solution up to the fifth approximation of the equation $\frac{dy}{dx} = y + x$ such that $y = 1$ when $x = 0$.

6. a) Find by Taylor's series, the value of y at $x = 0.1$ and $x = 0.2$ to five places of decimals from : $\frac{dy}{dx} = x^2 y - 1$ and $y = 1$ when $x = 0$.

   b) Apply Runge-kutta fourth order method to find an approximate value of y, when $x = 0.2$ given that:

$$\frac{dy}{dx} = x + y \text{ and } y = 1 \text{ when } x = 0.$$

7. a) Determine the poles and residue at each pole of the function:

$$f(z) = \frac{z^2}{(z+2)(z-1)^2}$$

b) Find the kind of singularities of the function

$$f(z) = \sin\frac{1}{1-z} \text{ at } z = 1$$

8. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1, \; for\,|x| < 1 \\ 0, \; for\,|x| > 1 \end{cases}$$

b) Solve the differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t \;, y(0) = 0, y(0) = 1$$

using Laplace transform.

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